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COMPOSITES THEORY AND PRACTICE

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FUZZY SETS - FATIGUE TESTS

In this study, the results of fatigue tests using fuzzy set methodology are considered. Cyclic loading causes damage, reducing the strength until the material can no longer sustain even service loading. The experiments demonstrate the scatter of results. Fuzzy set analysis has been proposed in order to estimate the uncertainty in the evaluation of stiffness and critical number of cycles corresponding to final fatigue damage. The results of fatigue-life tests using fuzzy set methodology are also considered when the experimental results are given in the form of a histogram. The fuzzy set and vertex method in conjunction with finite element (FE) computations are introduced to evaluate buckling loads.

Keywords: fuzzy logic, fuzzy sets, fatigue tests, composite plates

ZBIORY ROZMYTE - BADANIA ZMĘCZENIOWE

Przedstawiono analizę wyników badań zmęczeniowych z wykorzystaniem metodologii zbiorów rozmytych. Cykliczne obciążenie powoduje uszkodzenia, co zmniejsza wytrzymałość aż do momentu, gdy materiał nie może przenosić nawet obciążenia eksploatacyjnego. Eksperyment wykazuje znaczny rozrzut wyników. Zaproponowano zastosowanie zbiorów rozmytych do oszacowania niepewności w ocenie sztywności i krytycznej liczby cykli odpowiadającej zmęczeniowemu zniszczeniu. Rozważono także wykorzystanie zbiorów rozmytych do analizy wyników badań zmęczeniowych, gdy wyniki doświadczalne są podane w formie histogramu. Zbiory rozmyte i metoda vertex z wykorzystaniem metody elementów skończonych (MES) zostały wprowadzone do oceny obciążenia krytycznego.

Słowa kluczowe: logika rozmyta, zbiory rozmyte, badania zmęczeniowe, płyty kompozytowe

INTRODUCTION

Composite structures are usually made up of many layers. Each layer may have different thicknesses and fiber orientations. Experimental data, especially for fatigue tests, have ranges of scatter affected by variability in the material microstructure from one test specimen to another, of course, if we do not want to also mention the effects of stacking sequences etc. Typically, the variability in material parameters makes it difficult to accurately predict the response of structural components and significantly affects the reliability of designs, see e.g. monograph [1]. Therefore, commonly the probability theory is employed to characterize variability in material parameters. The material characteristics are treated as random variables with assumed probability density distributions. However, the critical review (see Yang et al. [2]) of the existing probabilistic descriptions of fatigue tests demonstrates evidently that: (i) there is no unique approach (or methodology) that may be used for the statistical characterization of fiber composite fatigue life data, (ii) there is a lack of a universal probability density distribution which may describe fatigue tests for composite materials, though it is commonly assumed that fiber reinforced materials follow Weibull distribution - see e.g. Whitney [3], Hahn & Kim [4], (iii) the construction of probability density distributions requires sufficient (commonly a great number) experimental data. Nevertheless, it should be emphasized that we do not know in advance anything about the nature of the material parameters, e.g., can they be treated as stochastic or non-stochastic parameters? The existence or lack of appropriate probability density distributions is a hypothesis only and the correctness of such a hypothesis can be verified on the basis of existing experimental data. Thus, for situations where the material parameter values are modeled as non-stochastic variables, the fuzzy set approach has been proposed as a better and more natural approach. The latter hypothesis is more general than previously mentioned. The scatter of properties has a different origin, but in general, it may be divided and classified in the following manner: (1) geometric properties (imperfections), (2) physical and mechanical properties, (3) environmental effects (exploitation), (4) technology understood in the sense of geometric dimensions but as an origin of local defects, a scatter of fiber directions etc.

Different kinds of methods are investigated for different types of uncertainties. Oskay and Fish [5] calibrated material properties in a deterministic fashion with the aid of genetic algorithms and gradient-based techniques. Jiang et al. [6] proposed the deterministic model to identify the uncertain elastic modulus of braided composites using modal data. Mustafa et al. [7] presented a probabilistic model for estimating the fatigue life of composite laminates using a high fidelity, multi-scale approach called M-LaF (Micromechanics based approach for Fatigue Life Failure). They developed a unified framework for the representation and quantification of uncertainty present in the fiber and matrix properties with the use of the Bayesian inference approach in order to calculate probabilistic composite fatigue failure. Chandrashekhar and Ganguli [8] performed probabilistic analysis using the Monte Carlo Simulation on a refined composite plate finite element model to calculate the statistical properties of the variation in modal parameters of a cantilever composite plate due to structural damage and material uncertainty.

For composite structures, due to the limited availability of test samples or research data, a various scatter of results (e.g. [9]) is observed. Moens and Hanss [10] presented an overview of the research activities on non-probabilistic finite element analysis and its application in the representation of parametric uncertainty in applied mechanics. Altmann et al. [11] introduced a fuzzy-probabilistic approach to assess the durability of strain-hardening cement-based composites. Karbhari and Stein [12] described the application of a fuzzy probabilistic approach to reflect the impact of the inherent uncertainty in determining the reinforcing fibers of polymer jackets for the seismic retrofit of columns. Rajmohan et al. [13] used grey fuzzy logic to optimize drilling parameters that minimizes the damage caused during drilling. Muc and Kędziora [14, 15] formulated the optimization problem as a λ-problem in which the maximum of the λ parameter was sought. Bohlooli et al. [16] presented a suitable model based on fuzzy logic to predict the compressive strength of inorganic polymers with seeded fly ash and rice husk bark ash.

Syamsiah Abu Bakar et al. [17] proposed the role of input selection using a neuro-fuzzy approach to predict the physical properties of degradable composites, namely the melt flow index and density. Vassilopoulos and Bedi [18] used an adaptive neuro-fuzzy inference system to model the fatigue behavior of a multi-directional composite laminate. Jarrah [19] proposed a hybrid neuro-fuzzy method that gave more accurate fatigue life predictions of unidirectional glass fiber/epoxy composite laminates. Rassbach [20] developed a model based on the mathematical methods of fuzzy-logic which can describe the plasto-mechanical behavior of functionally gradient materials during the deformation process.

Therefore, in this way the impreciseness (uncertainty) of the experimentally obtained data in our approach is described using fuzzy sets theory. The application of fuzzy set methodology is presented in refs. [14, 15, 21-24]. It is possible to construct a membership function for the measured value, hence we intend to adjust to the membership function representation. With the use of the above-mentioned construction, it is possible to build a Fuzzy Knowledge Base or probabilities for a given set of experimental data. We propose the fuzzy set method for the correct and concise evaluation of various fatigue properties (e.g. stiffness). We demonstrate the applicability of fuzzy set theory and the vertex method to analyze the limit load carrying capacity of structures in conjunction with finite element (FE) computations.

FUNDAMENTAL DEFINITIONS - REPRESENTATIONS OF FUZZY SETS

The data "integer less than 10" is the definition of characteristic function Π_A and is represented in the following way:

$$\Pi_A : IN \to \{0,1\}$$
$$\Pi_A(\eta) = \begin{cases} 1, & \text{if less than } 10 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

that yields a value of 1 for each element of space *IN* that belongs to set *A* and a value of 0 for each element that does not. The above representation is commonly called a crisp set. However, this concept cannot be used directly as we intend to characterize the typical property for composite materials, e.g. the failure of CFRP under tension occurring as tensile strain ε_x is equal to the ultimate value of 0.015. The characteristic function of this set is depicted in Figure 1a. For the three-dimensional analysis, all the components of the strain tensor can be evaluated in a similar manner. A problem arises if the linguistic term "failure under tension" has to be described. The value of a failure strain in composites depends on various parameters such as the fiber and matrix materials, loading conditions, porosity, environmental effects, etc. It is well-known that from the micromechanical point of view that failure starts from microcracks in the matrix for strain values much lower than the $\varepsilon_{allowable}$ value. In addition, the $\varepsilon_{allowable}$ value is usually an average value characterizing rather a scatter of a random number of macrocracks appearing at the $\varepsilon_{allowable}$ strain level. Therefore, for some specimens one can observe the final (macroscale) failure as ε_x is equal to 1.1 $\varepsilon_{allowable}$ or to 0.9 $\varepsilon_{allowable}$. As may be seen, the variability (fuzziness) is taken to be equal to ±10%, which falls within typical ranges of scatter in experimental data for static tests. A possible solution to this problem is to generalize the definition of the characteristic function in a way that it yields values from interval

[0, 1] and not just the two values of set {0, 1}. This leads to the notion of a fuzzy set. Fuzzy set μ of X is a function that maps space X into the unit interval, i.e.:

$$\mu : X \to [0,1] \tag{2}$$

Value $\mu(x)$ denotes the membership function of x to fuzzy set μ. Figure 1b shows (subjectively defined) a membership function of fuzzy set μ describing the linguistic meaning of the term “failure under tension”. The use of fuzzy sets to formally represent vague data is often done in an intuitive way because in many applications there is no model that provides a clear interpretation of the membership degrees, though we want or we try to base it on various experimental data.

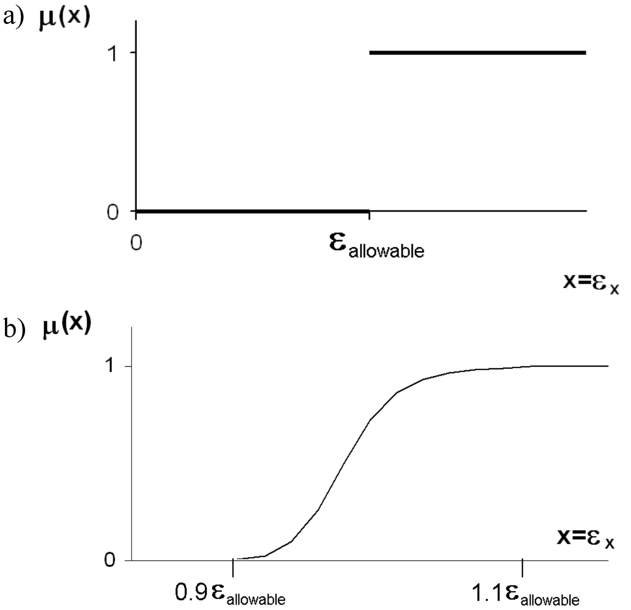

Fig. 1. Representation of term “failure under tension”: a) crisp set, b) fuzzy set

Rys. 1. Graficzne przedstawienie określenia “zniszczenia podczas rozciągania”: a) zbiór ostry, b) zbiór rozmyty

The application of fuzzy methodologies requires the knowledge of the membership functions of fuzzy quantities. In general, fuzzy numbers are sets that represent numeric quantity. It can be done in a variety of ways. Of course, there are different possibilities to determine and represent membership functions characterizing a fuzzy set. If subspace S contains only a finite number of elements, fuzzy set μ of X will be defined by specifying for each element $x \in X$ its membership degree $\mu(x)$. If the number of elements is very large or a continuum is chosen for X, then $\mu(x)$ can be better defined by a function that can use parameters which are adapted to the actual modeling problem. For instance, if we want to represent the term “Young’s modulus is equal to 200 GPa” in the sense of a fuzzy set having a finite amount of experimental data, we can select one of the different representations given in Figure 2.

One of the possible fuzzy set representations was presented there. Another approach is the so-called horizontal representation of fuzzy sets. This is introduced by using their α-cuts instead of membership functions μ(x) which are called vertical representations.

Let $\mu \in F(x)$ and $\alpha \in [0, 1]$. The set is called the α-cuts of μ:

$$[\mu]_\alpha = \{x \in X | \mu(x) \geq \alpha\} \tag{3}$$

Let μ be the triangular function on IR given in Figure 3. The α-cuts of μ are in this case defined as follow:

$$[\mu]_\alpha = \begin{cases} [a + \alpha \cdot (m-a), b - a \cdot (b-m)] & \text{if } 0 < \alpha \leq 1 \\ IR & \text{if } \alpha = 0 \end{cases} \tag{4}$$

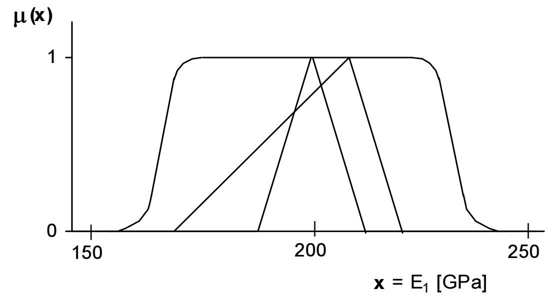

Fig. 2. Various fuzzy representations of term “Young’s modulus is equal to 200 GPa”

Rys. 2. Różne typy funkcji przynależności opisujące termin „moduł Younga równa się 200 GPa”

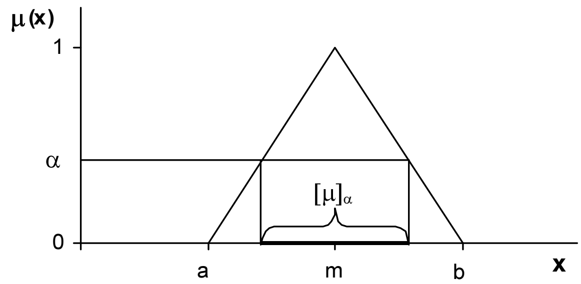

Fig. 3. Definition of α-cuts

Rys. 3. Definicja przekrojów α

VERTEX METHOD

Let us introduce N fuzzy parameters describing the material or geometric parameters of the considered composite structure. The membership functions are discretised using several α-cuts - Eq. (4). Considering the left and right end points of α-cut intervals $[\mu]_\alpha$ (see Figure 3) for all the fuzzy parameters, one can find the total number of combinations $N_{c/\alpha}$ per α-cut in the following form:

$$N_{c/\alpha} = \begin{cases} 2^N & \text{for } 0 \leq \alpha < 1 \\ 1 & \text{for } \alpha = 1 \end{cases} \tag{5}$$

An output response denoted by p is an unknown function of input fuzzy parameters x_i (i = 1, 2, …, N), so that:

$$p = f(x_1,...,x_N) \tag{6}$$

Using the α-cut concept combined with binary representation (5) of fuzzy parameters x_i (i = 1,2,…,N) relation (6) can be rewritten in the abbreviated form:

$$p = f(C_{\lambda,j}); \quad j = 1, 2, ..., N_{c/\alpha} \tag{7}$$

Since output response p as a function of fuzzy parameters is a fuzzy set, the corresponding interval in p is obtained from relation (6):

$$[p_\alpha^L, p_\alpha^R] = [\min_{\lambda,j} f(C_{\lambda,j}), \max_{\lambda,j} f(C_{\lambda,j}); \\ \lambda \geq \alpha, \ j = 1, 2, ..., N_{c/\alpha} \tag{8}$$

As may be seen, relation (8) allows one to obtain a scatter of the output parameters and then to build the appropriate probability distributions and reliability functions by a sweep of α-cut at different possibility levels.

In order to conduct the computations and to evaluate the upper and lower boundaries of output response (8), it is necessary to outline the deterministic method of the definition of function f given in Eq. (6). It can be defined in a purely analytical way or alternatively in a purely numerical way. As may be noticed, the vertex method resembles here the Monte Carlo simulation method where the output response also has a deterministic, and therefore unique form. Function f existing in Eq. (6) may describe an arbitrary failure criterion for composites, e.g. buckling, delamination, first-ply-failure etc., whereas symbol p denotes the corresponding value of the failure load.

FUZZY LOGIC ANALYSIS OF EXPERIMENTAL DATA

There are, generally, two kinds of fuzzy sets:

1. numbers that represent an approximate numeric quantity
2. qualifiers that characterize open-ended concepts; these sets provide the framework for describing unbounded concepts (or concepts that are theoretically unbounded).

Fuzzy numbers create an important class of fuzzy sets that are spread around a central value. The generalized approximation curve indicated by around or close concepts, produces a membership function for the fuzzy assertion representing a fuzzy space of all the numbers that are around or close to a central value.

In general, there are five important classes of fuzzy contours describing fuzzy numbers: (i) triangular curves, (ii) trapezoidal (shouldered) curves, (iii) π (pi) curves, (iv) β (beta) curves and (v) Gaussian curves. They are defined e.g. in paper [14].

For composite materials, microscopic defects are coalesced and they grow as micro-cracks in structural materials throughout the various loading histories. The membership for failure is directly related to the damage variable both for static and fatigue loading conditions. In the research presented in paper [24], the spatial non-uniformity of material properties at the microscopic level is to be taken into account from experimental data obtained during fatigue tests conducted for plies oriented at 0°, 45° and 90° in tension and compression. When processed, this information will be represented by the lower and upper boundaries of the stiffness degradation, i.e. as modulus $E(n)$ versus the number of cycles n relationships. $E(n)$ denotes the longitudinal elastic modulus for 0° fiber orientations, transverse elastic modulus for 90° fiber orientations and shear modulus for 45° fiber orientations. These sets of lower and upper boundaries will be available independently for 0°, 45° and 90° orientations. With the use of the fuzzy logic methodology described in this paper and [14, 22-24], the upper and lower boundaries of stiffness $E(n)$ are plotted in Figure 4. These figures illustrate the form of diagrams obtained for fibers oriented at 0° for specimens subjected to tension. N_f denotes fatigue life.

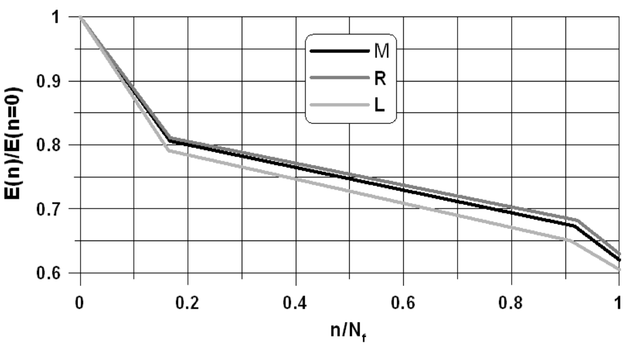

Fig. 4. Degradation of laminate (glass fiber/epoxy resin) stiffness vs. number of cycles (mean values - M, upper - R and lower - L bounds, respectively)

Rys. 4. Degradacja sztywności laminatu (włókna szklane/żywica epoksydowa) w zależności od liczby cykli (wartości średnie - M, kres górny - R, kres dolny - L)

Determining the membership functions is difficult as Norwich et al. [25], Dombi [26] point out. Thus, the first attempt at or trial in building the membership functions can be based on statistical data. However, it is to be noted that not all fuzzy quantities have statistical data to define their membership functions. If statistical data exists, the membership function can be determined as follows:

$$\mu(x) = \lambda\rho(x), \quad \lambda = 1/\max[\rho(x)] \tag{9}$$

where $\rho(x)$ is a probability density function or its estimate derived from the histogram of feature x used to define the fuzzy set.

Let us consider the results of fatigue-life tests using fuzzy set methodology, when the experimental results are given in the form of a histogram. In each part of the histogram, we intend to adjust to the membership function representation in the following manner (see also Figure 5):

1. to find the mean value of fatigue life $\overline{N}_{f(j)}$ in the i-th interval
2. to prescribe that the value of the membership function corresponding to the mean value of the fatigue lives is equal to 1
3. to assume that the values of the membership function at the ends (i.e. $N_f(i)^L$ and $N_f(i)^R$) of the i-th interval are equal to 0
4. to determine the value of the membership function for the given experimental data $N_{f(i)}^{\exp}$

$$\mu(N_{f(i)}^{\exp}) = \frac{N_{f(i)}^{R(L)} - N_{f(i)}^{\exp}}{N_{f(i)}^{R(L)} - \overline{N}_{f(i)}} \tag{10}$$

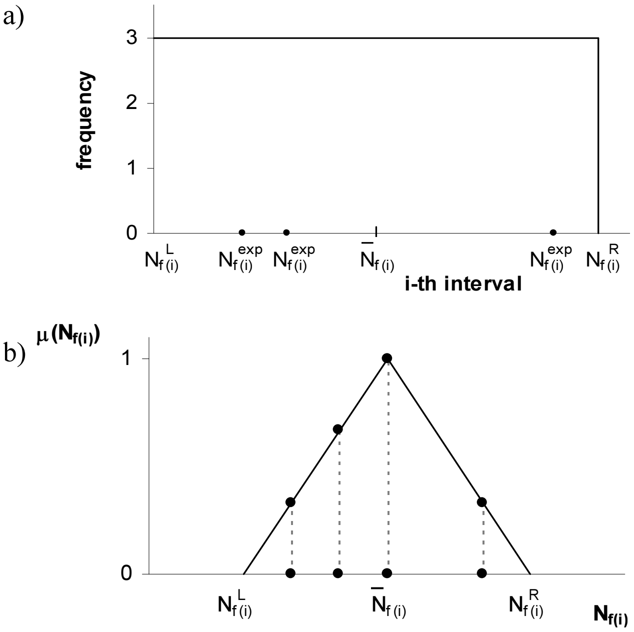

Fig. 5. Experimental results as: a) histogram, b) membership function

Rys. 5. Wyniki badań w postaci: a) fragment histogramu, b) funkcji przynależności

More information on using the above-mentioned construction can be found in [15]. With the use of the above-mentioned construction, it is possible to build a Fuzzy Knowledge Base or probabilities for the given set of experimental data.

FUZZY SET ANALYSIS OF LIMIT LOAD CARRYING CAPACITY - EFFECTS OF SCATTER OF MECHANICAL PROPERTIES

For composite structures, the concept of the limit load carrying capacity (LLCC) was introduced by Muc et al. [27]. Briefly speaking, the fundamental idea of LLCC is based on determining the lower limit envelopes of different failure loads corresponding to the analyzed composite structure subjected to prescribed loading and boundary conditions and having a uniquely defined laminate topology. As different failure loads we understand failure loads corresponding to various failure modes encountered in the analysis of composite structures, i.e. delaminations, matrix cracking, global or local buckling, fiber debonding etc. In detail, in the present work one of several types of failure modes are considered: global buckling.

To evaluate global buckling loads, numerical studies are performed on a square plate under compression. The numerical FE analysis is carried out in the elastic geometrically linear range only, with the use of four-node quadrilateral shell elements (NKTP 32) employing the first order transverse shear deformation plate/shell theory. The geometric and material characteristics are the following: E_x = 280 GPa, E_y = 12 GPa, G_{xy} = 7 GPa, G_{xz} = 0.6 G_{xy}, G_{yz} = G_{xy}, ν_{xy} = 0.28, t/a = 0.1, where t is the plate thickness and a is the plate length, respectively. The vertex method is introduced herein to evaluate buckling loads in a numerical manner. Four parameters have been considered as fuzzy variables: total thickness t, Young's moduli E_x, E_y and Kirchhoff modulus G_{xy}.

In the present study, it is assumed that the membership functions of the fuzzy parameters are triangular as shown schematically in Figure 3 (see also Eq. (4)) where:

$$m = \frac{a+b}{2},\ a = 0.9 \cdot m\ b = 1.1 \cdot m \tag{11}$$

and m is an average value for each of the fuzzy parameters, for instance it can be evaluated from the experimental data. As it may be seen from relation (11), the variability (fuzziness) is taken to be equal to ±10%, which falls within typical ranges of scatter in experimental data for static tests.

The distributions of buckling pressures versus the angle of fiber orientations at α = 0, 0.5 and 1.0 are plotted in Figure 6. The upper and lower boundaries (α = 0 and 0.5) of the curves drawn for α = 1.0 are not symmetric. Interval (8) is strongly dependent on the fiber orientations as well as on the wave number in buckling.

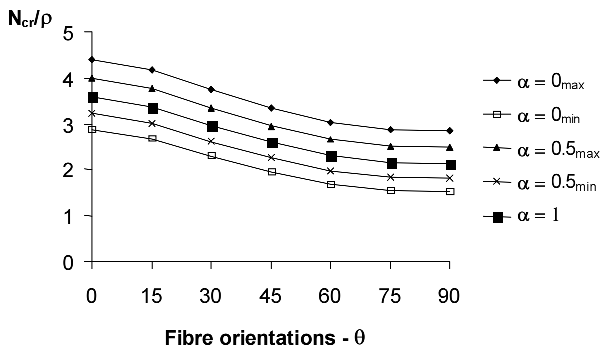

Fig. 6. Distributions of buckling loads for compressed angle-ply plates

Rys. 6. Bezwymiarowe obciążenia krytyczne dla ściskanych płyt wykonanych z laminatów kątowych

CONCLUDING REMARKS

The present study is a practical tool for engineering activities dealing with evaluating the degradation of real material structure. As it remains an open problem, it is connected with the total number of uncertain parameters that should be considered in order to describe the real behavior of engineering structures with an acceptable accuracy. However, it can be solved for each individual problem only.

In many practical situations, the realistic modeling of phenomena necessitates the use of irregularly shaped, nonlinear membership functions, concave or convex, continuous or discontinuous, especially since a designer intends to obtain a better approximation of the model variables. Nevertheless, the previously enumerated membership functions, particularly linear, triangular or trapezoidal are commonly used in fuzzy set applications, mainly for reasons of computational simplicity. The linear membership functions are also used in the mathematical formulations of optimization problems in a fuzzy environment. By using an identical methodology, it is possible to consider delaminations, first-ply-failure and buckling problems (in conjunction with FE analysis) for arbitrary types of laminated composite structures having different types of material and geometric imperfections.

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